

# Subgrid Technology for High Resolution Modeling Nearshore – a straw man proposal

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Chesapeake Bay shallow water modeling  
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5/26-27,2012

# What is subgrid modeling?

- Subgrid-scale modeling refers to the representation of important small-scale physical processes that occur at length-scales that cannot be adequately resolved on a computational mesh.
- Example: In Large-Eddy Simulation of turbulence, subgrid-scale (SGS) modeling is used to represent the effects of unresolved small-scale fluid motions (small eddies, swirls, vortices) in the equations governing the large-scale motions that are resolved in computer models.
- The formulation of physically realistic SGS models requires understanding of the physics and the statistics of scale interactions in hydrodynamic turbulence, and is an open research question owing to the fact that turbulence remains an unsolved problem in classical physics.

# Why subgrid technology can be used in nearshore dynamic modeling?

- The nearshore shallow than 2 m, look like a ribbon; it is a boundary layer
- Very dynamic, relatively narrow
- Need a high resolution to resolve complicated dynamics

- In the shallow water where friction is dominated, there is a governing equation which will allow dynamic interpolation of fine-scale velocity associated with high resolution bathymetry data without resort to solving full equation.

Can be achieved efficiently!!!!

# Differential conveyance formulation on a sub-grid scheme

- A simplified 2D depth averaged momentum equation:

$$\frac{DU}{Dt} + g \frac{\partial \zeta}{\partial x} + cf \frac{U \|U\|}{h} = 0$$

Where  $cf = g/C_z^2$  or  $cf = g n^2 / h^{1/3}$

- $\longrightarrow$   $\frac{DU}{Dt} + g \frac{\partial \zeta}{\partial x} + g \frac{U \|U\|}{\Omega^2} = 0$  Within each time step, the main balance can be the last two terms if friction dominates
- 
- 

Where  $\Omega = \sqrt{\frac{gh}{cf}}$  is conveyance, and  $\Omega = C_z \sqrt{h}$  or  $\Omega = \frac{h^{2/3}}{n}$  respectively

- In 2D sub-grid, if for each pixel  $\frac{cf_j}{gh_j} \|u_j\|^2 = \text{constant}$  (for  $\zeta_x = \text{constant}$ )  $\rightarrow \left( \frac{\|u_j\|}{\Omega_j} \right)^2 = \text{constant} \forall j$

Introduce cell average velocity U where

$$\|U\| = \frac{\sum_{j=1}^J h_j \|u_j\|}{\sum_{j=1}^J h_j}$$

- Then

$$\boxed{\frac{\|u_j\|}{\Omega_j} = \frac{\|U\|}{\Omega} \forall j \quad \text{or} \quad \|u_j\| = \Omega_j \frac{\|U\|}{\Omega}}$$

where  $\Omega = \frac{\sum_{j=1}^J h_j \Omega_j}{\sum_{j=1}^J h_j}$

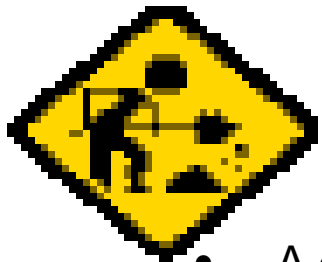
## Note

1. The subgrid velocities  $u_j$  can be determined from  $|U|$  calculated from base grid with efficiency
2. With the assistance of high resolution bathymetry data, the subgrid velocities  $u_j$  can reflect the variation of bottom friction and effect of sediment characteristics
3. The sub-grid can be done for structured as well as for unstructured grid

# Foundation of “Subgrid Modeling” by Casulli and Stelling

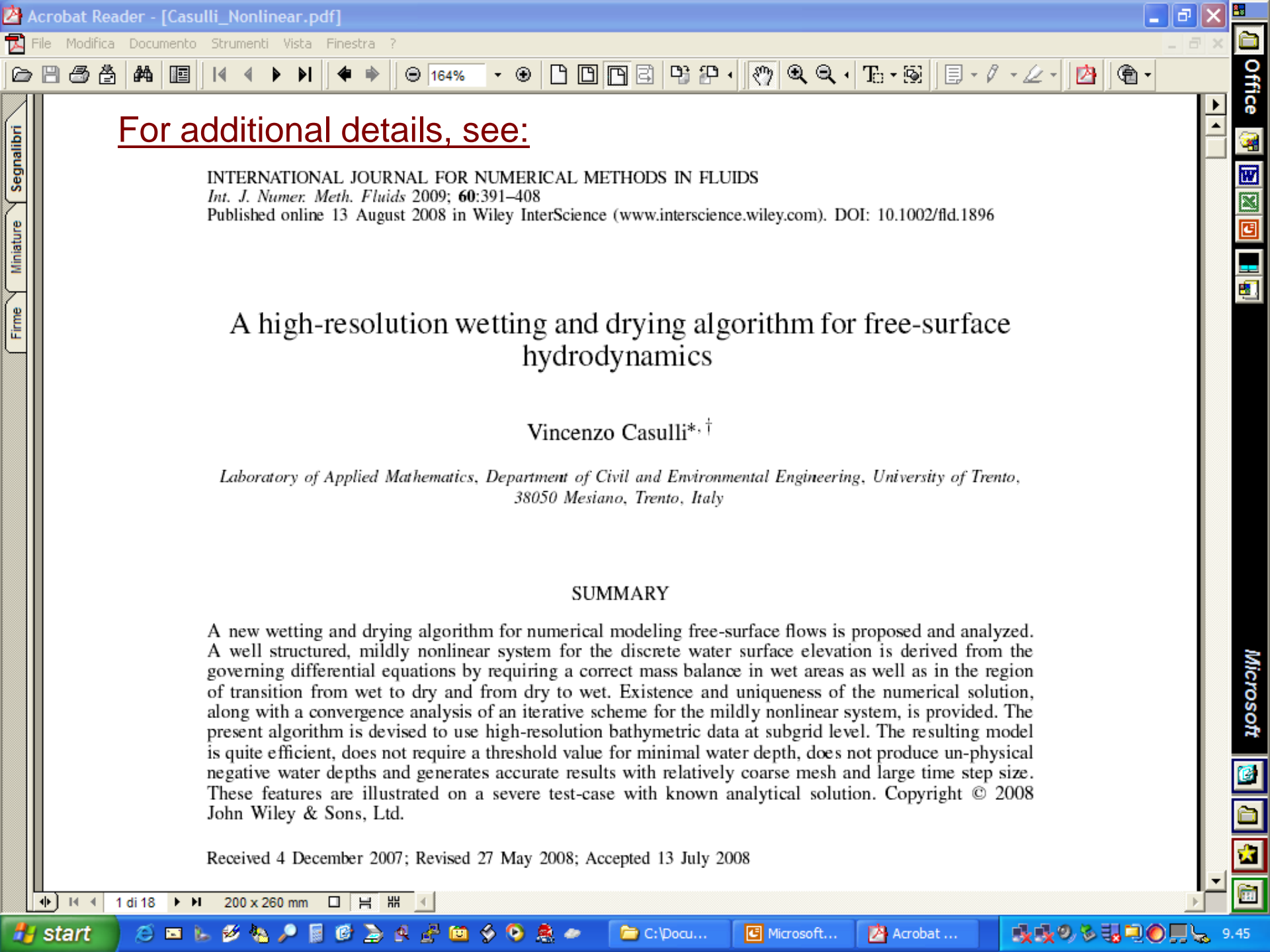
(Casulli V. and Stelling, G. S International Journal for Numerical Methods in Fluids, DOI:10.1002/fld.2361)

- *The availability of super computing power, solid and stable semi-implicit methods, structured or flexible unstructured grids are useful tools but, alone, are still insufficient to faithfully account for complex topographic features.*
- *The availability of detailed bathymetry data within a coarse grid model can and should be used to further improve a model accuracy*



**The Key elements for the sub-grid modeling GS (2010) are:**

- A differential form of conveyance formulation implemented on a sub-grid scheme
- Nonlinear semi-implicit solver for surface elevation



For additional details, see:

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS

*Int. J. Numer. Meth. Fluids* 2009; **60**:391–408

Published online 13 August 2008 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.1896

## A high-resolution wetting and drying algorithm for free-surface hydrodynamics

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*Laboratory of Applied Mathematics, Department of Civil and Environmental Engineering, University of Trento,  
38050 Mesiano, Trento, Italy*

### SUMMARY

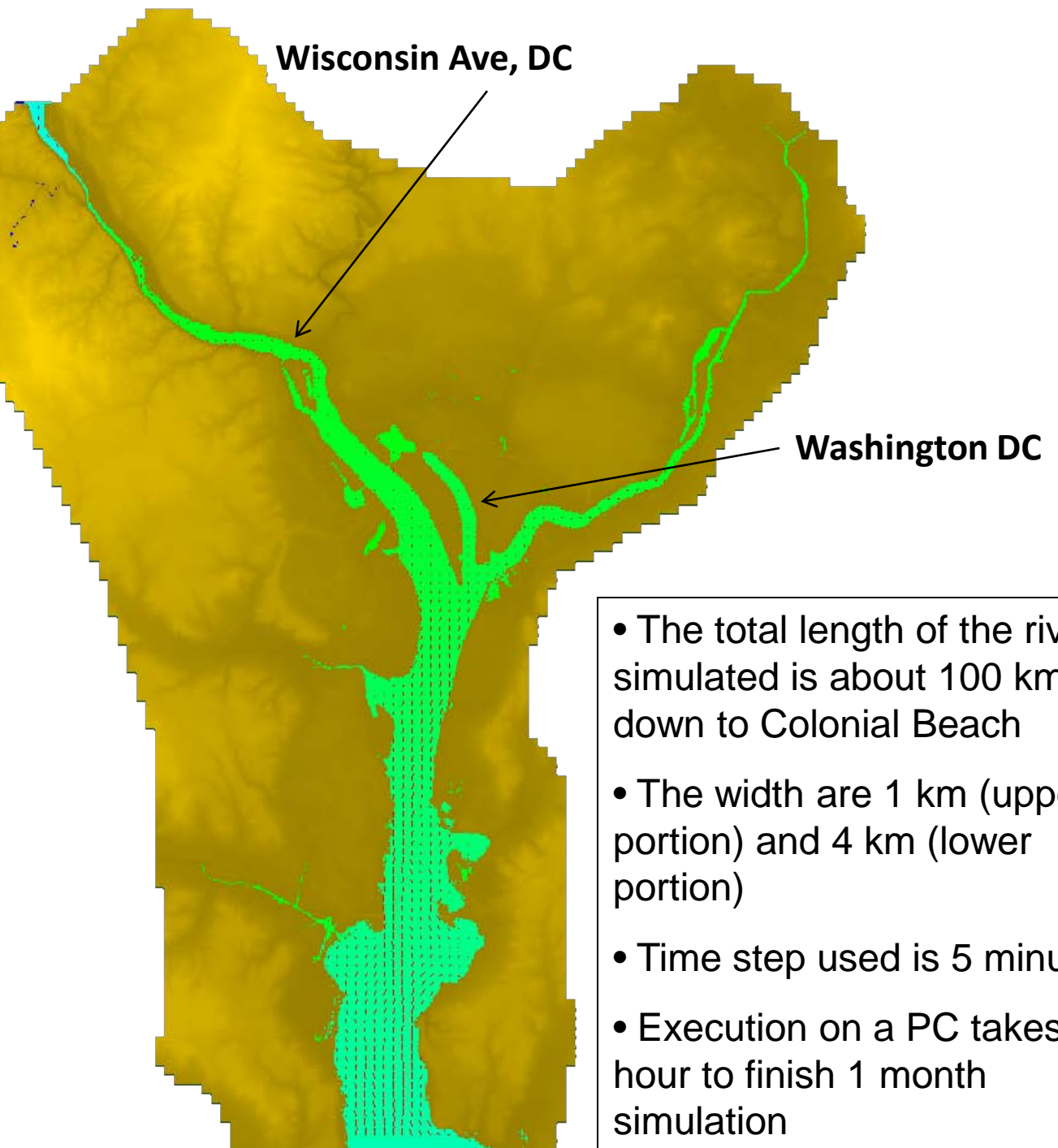
A new wetting and drying algorithm for numerical modeling free-surface flows is proposed and analyzed. A well structured, mildly nonlinear system for the discrete water surface elevation is derived from the governing differential equations by requiring a correct mass balance in wet areas as well as in the region of transition from wet to dry and from dry to wet. Existence and uniqueness of the numerical solution, along with a convergence analysis of an iterative scheme for the mildly nonlinear system, is provided. The present algorithm is devised to use high-resolution bathymetric data at subgrid level. The resulting model is quite efficient, does not require a threshold value for minimal water depth, does not produce un-physical negative water depths and generates accurate results with relatively coarse mesh and large time step size. These features are illustrated on a severe test-case with known analytical solution. Copyright © 2008 John Wiley & Sons, Ltd.

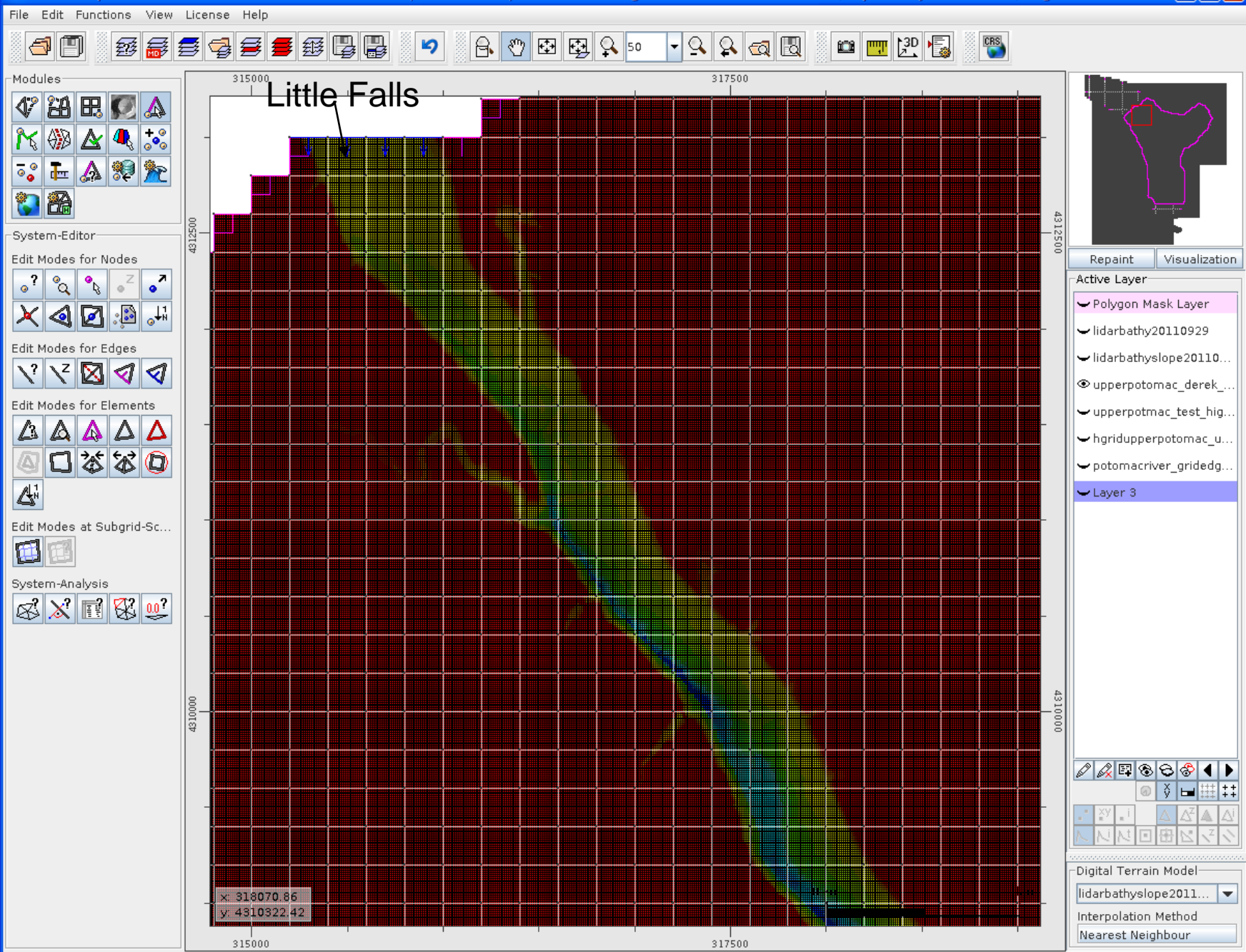
Received 4 December 2007; Revised 27 May 2008; Accepted 13 July 2008

# Example for using very high resolution sub-grid in Inundation modeling

## Preliminary Applications

of 'Sub-grid' technology in the Upper Potomac River (including Washington DC) with river discharge





File Edit Functions View License Help

Modules

System-Editor

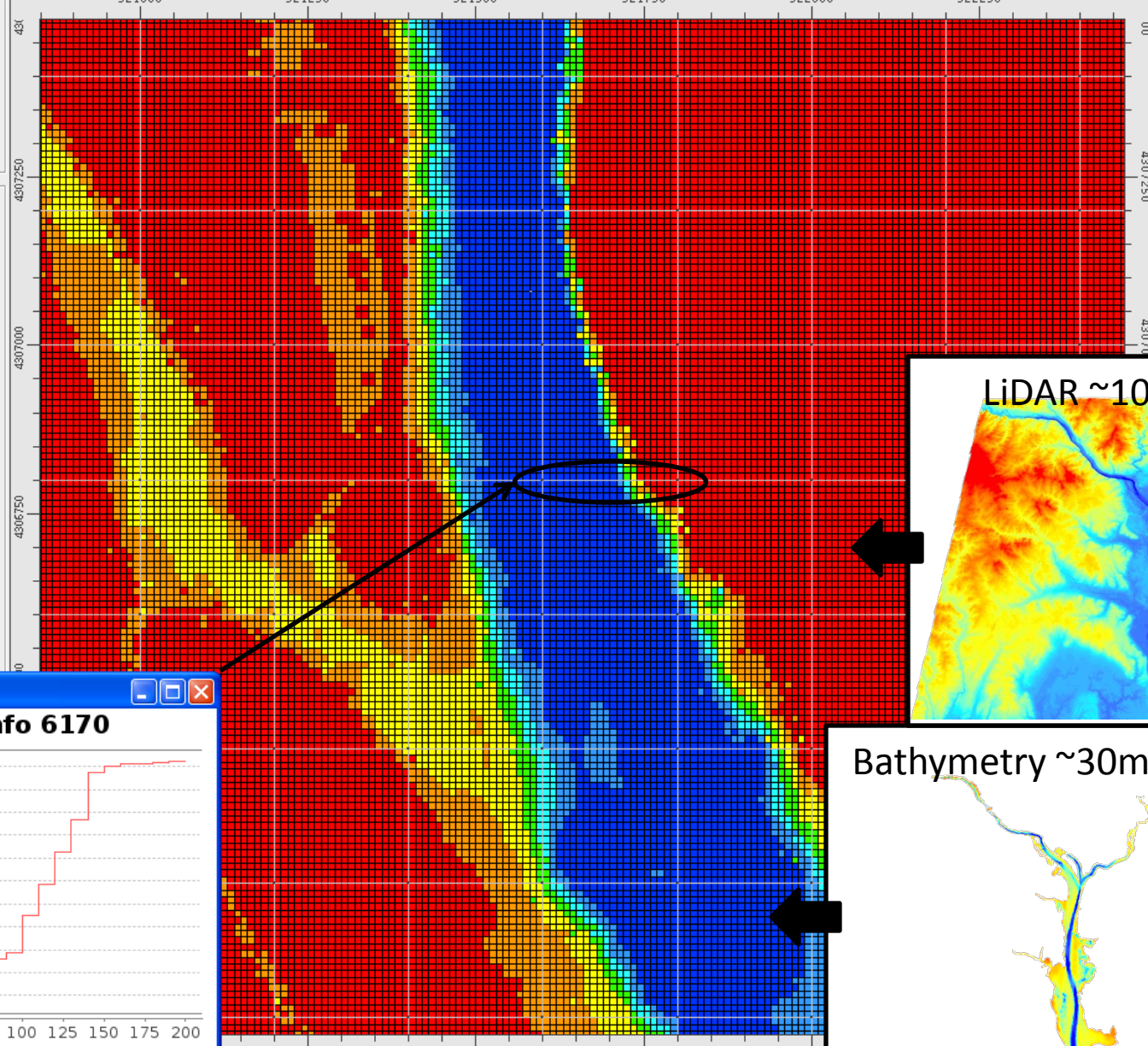
Edit Modes for Nodes

Edit Modes for Edges

Edit Modes for Elements

Edit Modes at Subgrid-Sc...

System-Analysis



321000 321250 321500 321750 322000 322250

4307000 4307250 4307500 4307750 4308000

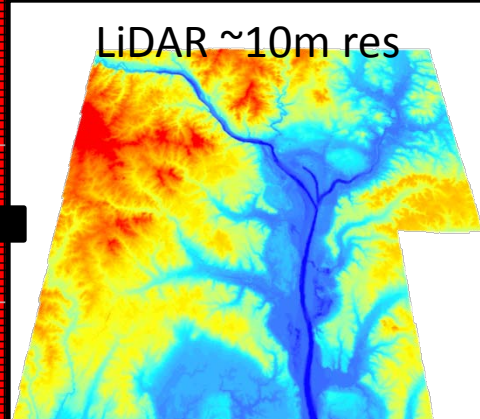
00 4307250 4307000

Repair Visualization

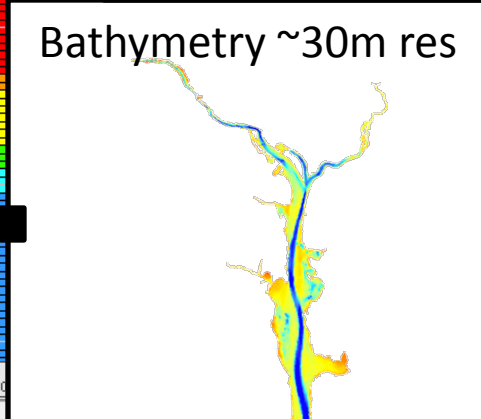
Active Layer

- Polygon Mask Layer
- lidarbathyslope20111...
- upperpotomac\_derek\_...

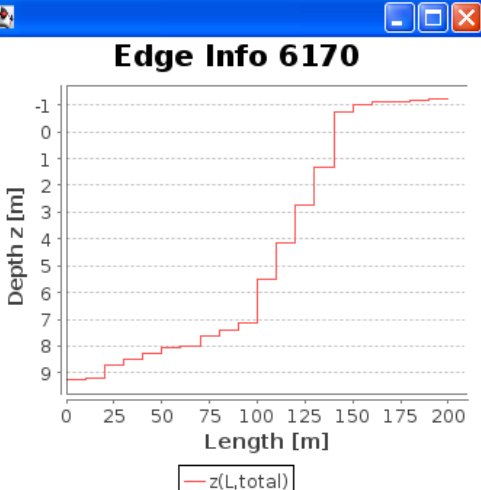
LiDAR ~10m res



Bathymetry ~30m res



Edge Info 6170



Depth z [m]

Length [m]

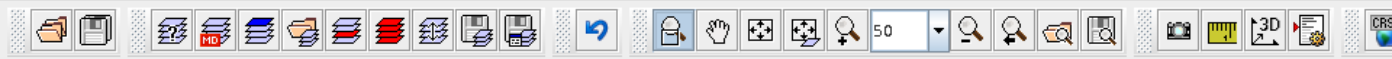
z(L,total)

rain Model

ep2

on Method

ighbour



Modules

System-Editor

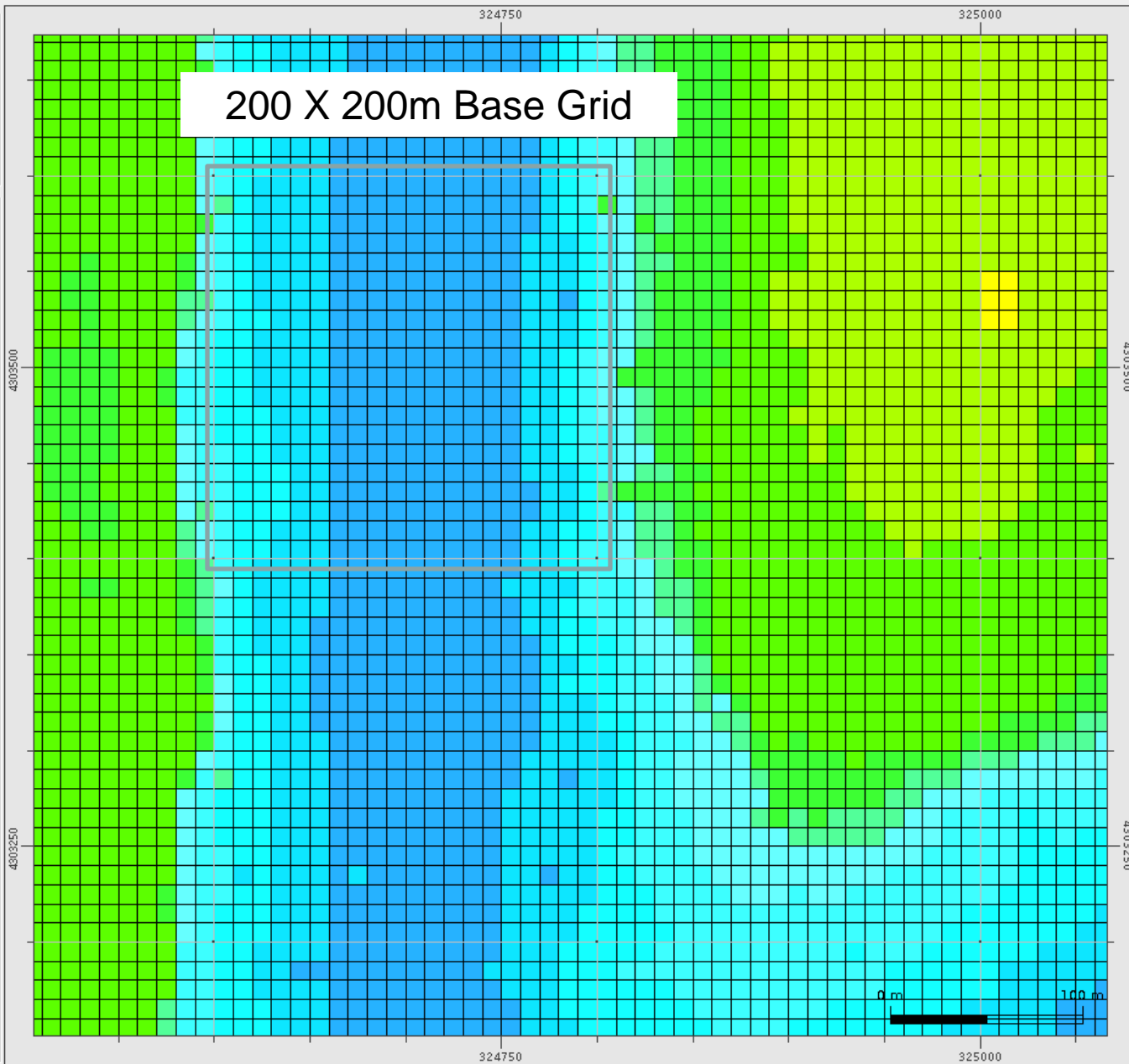
Edit Modes for Nodes

Edit Modes for Edges

Edit Modes for Elements

Edit Modes at Subgrid-Sc...

System-Analysis



Repair Visualization

Active Layer

- └ Polygon Mask Layer
- └ lidarbathyslope20110...
- └ upperpotomac\_derek\_...
- └ upperpotomac\_test\_hig...
- └ hgridupperpotomac\_u...
- └ potomacriver\_gridedg...
- └ Layer 3
- └ upperpotomac\_derekA...
- └ hgridupperpotomacwit...
- └ upperpotomac\_test2
- └ upperpotomac\_derek\_...
- └ upperpotomac\_derek\_...
- └ Layer 4
- └ Layer 5

Digital Terrain Model

lidarbathyslope2011...

Interpolation Method

Nearest Neighbour

# Washington DC Floods

1889



1936



1942

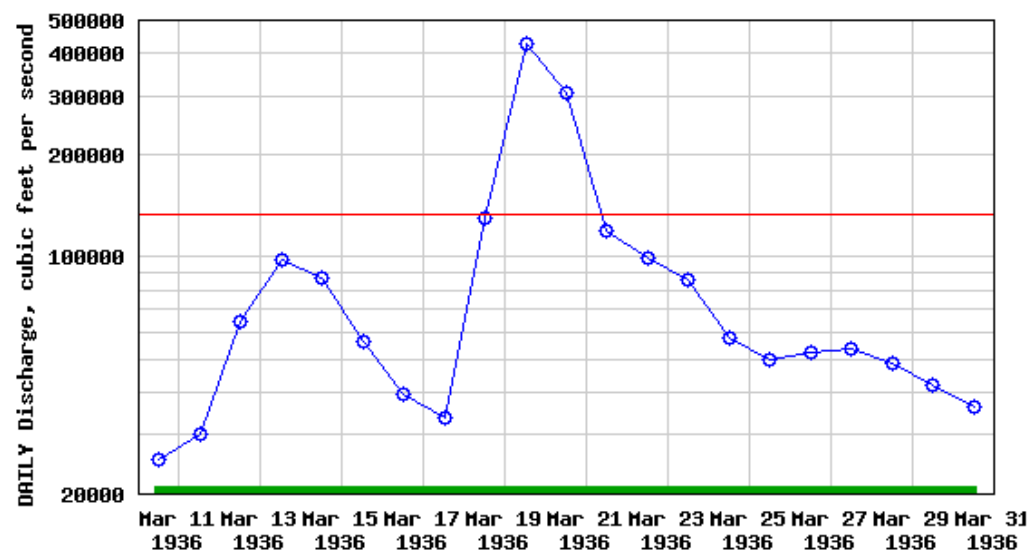


2006



# Discharge, cubic feet per second

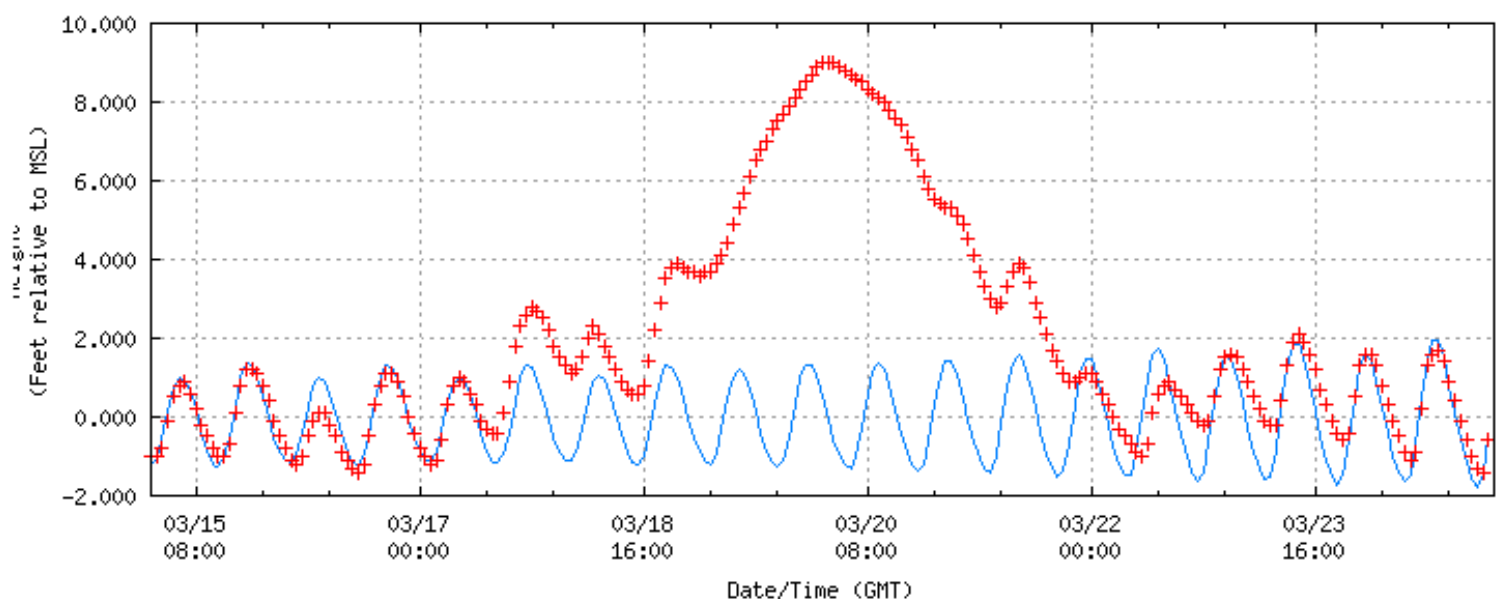
USGS 01646500 POTOMAC RIVER NEAR WASH, DC LITTLE FALLS PUMP STA



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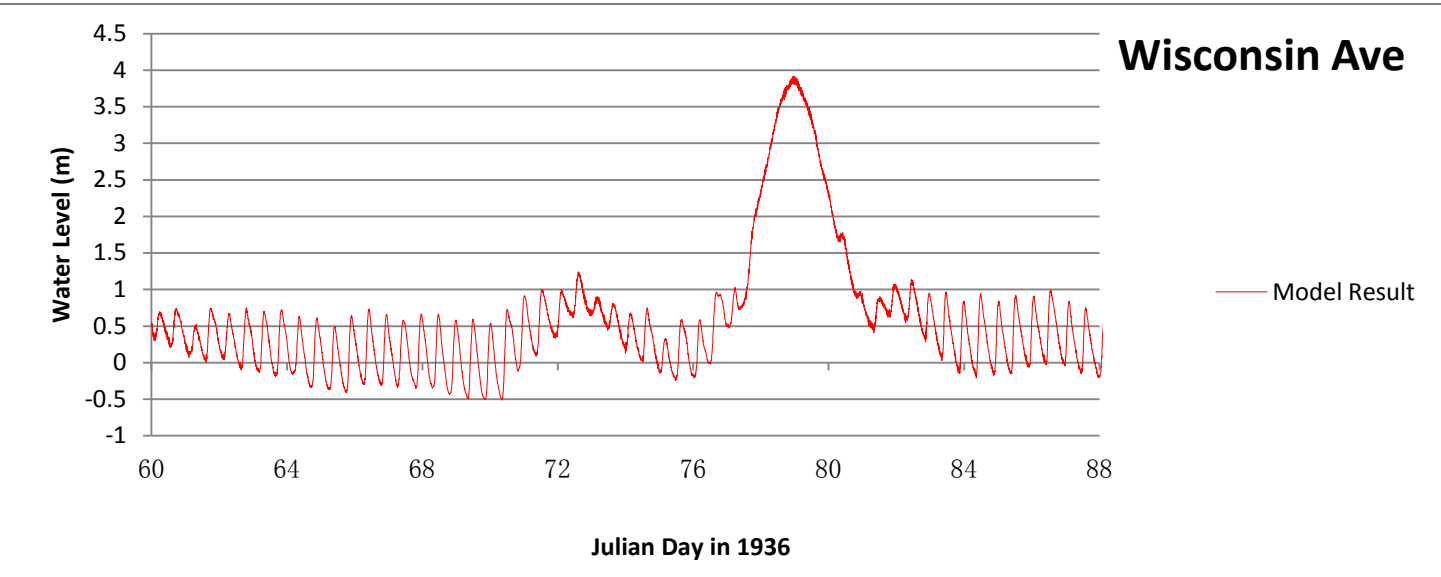
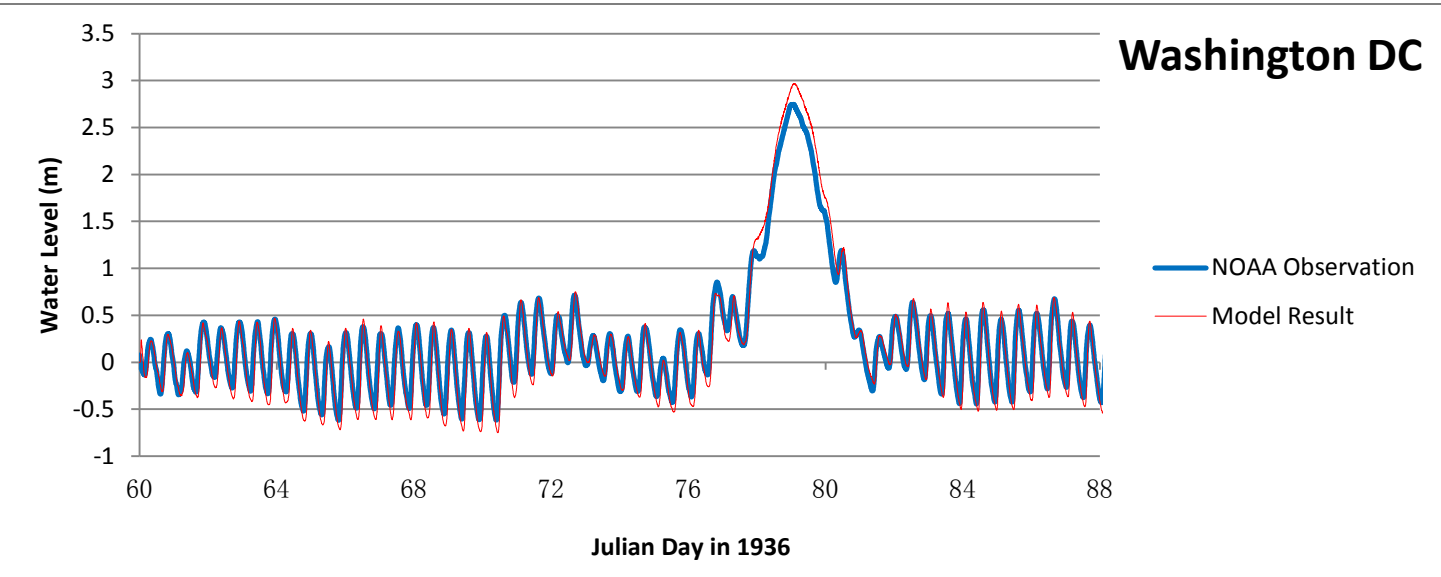
NOAA/NOS/CO-OPS  
 Verified Water Level vs. Predicted Plot  
 8594900 Washington, DC  
 from 1936/03/15 - 1936/03/24



# Configuration of Inundation Simulation for 1936 Great Flood

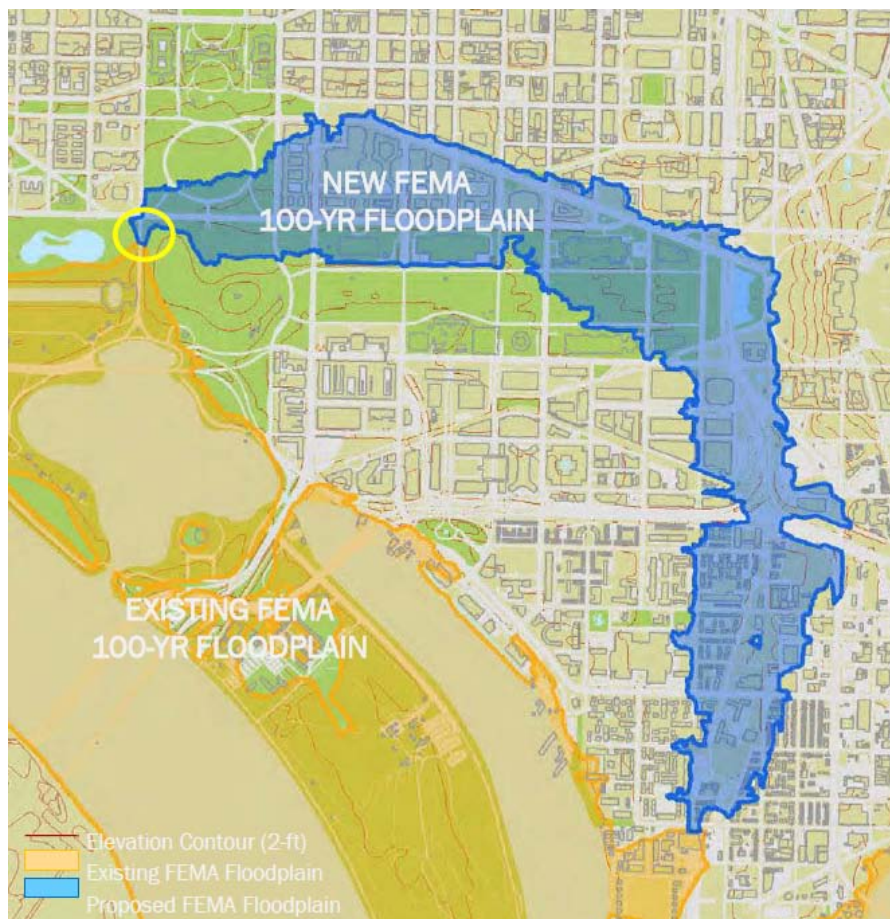
## Parameters:

- Constructed tidal boundary condition using observation data from Washington DC accounting for a 54 minute phase shift
- Coupled river discharge inputs from Potomac River at Little Falls, MD, and Anacostia River branch at Hyattsville, MD, and Riverdale, MD.
- Cz Value: 100
- Radiation open boundary condition:  $TRX=100$  (mildly radiation bc)

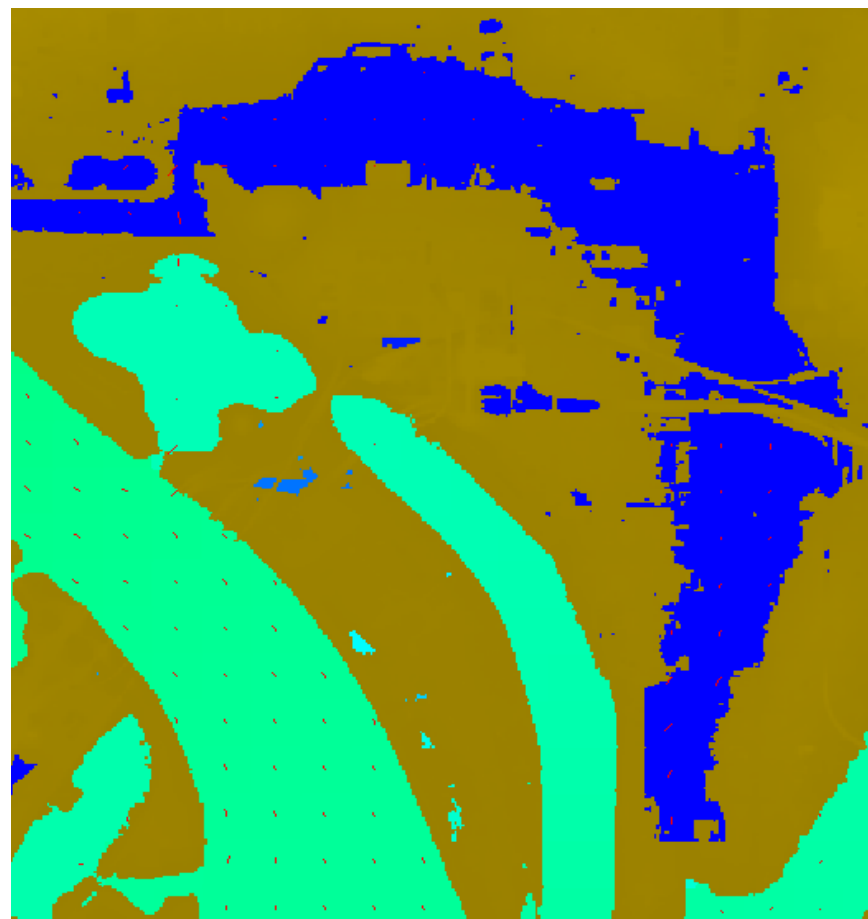


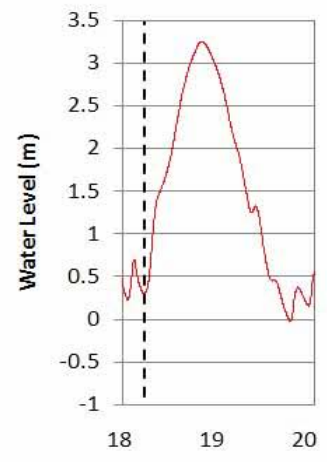
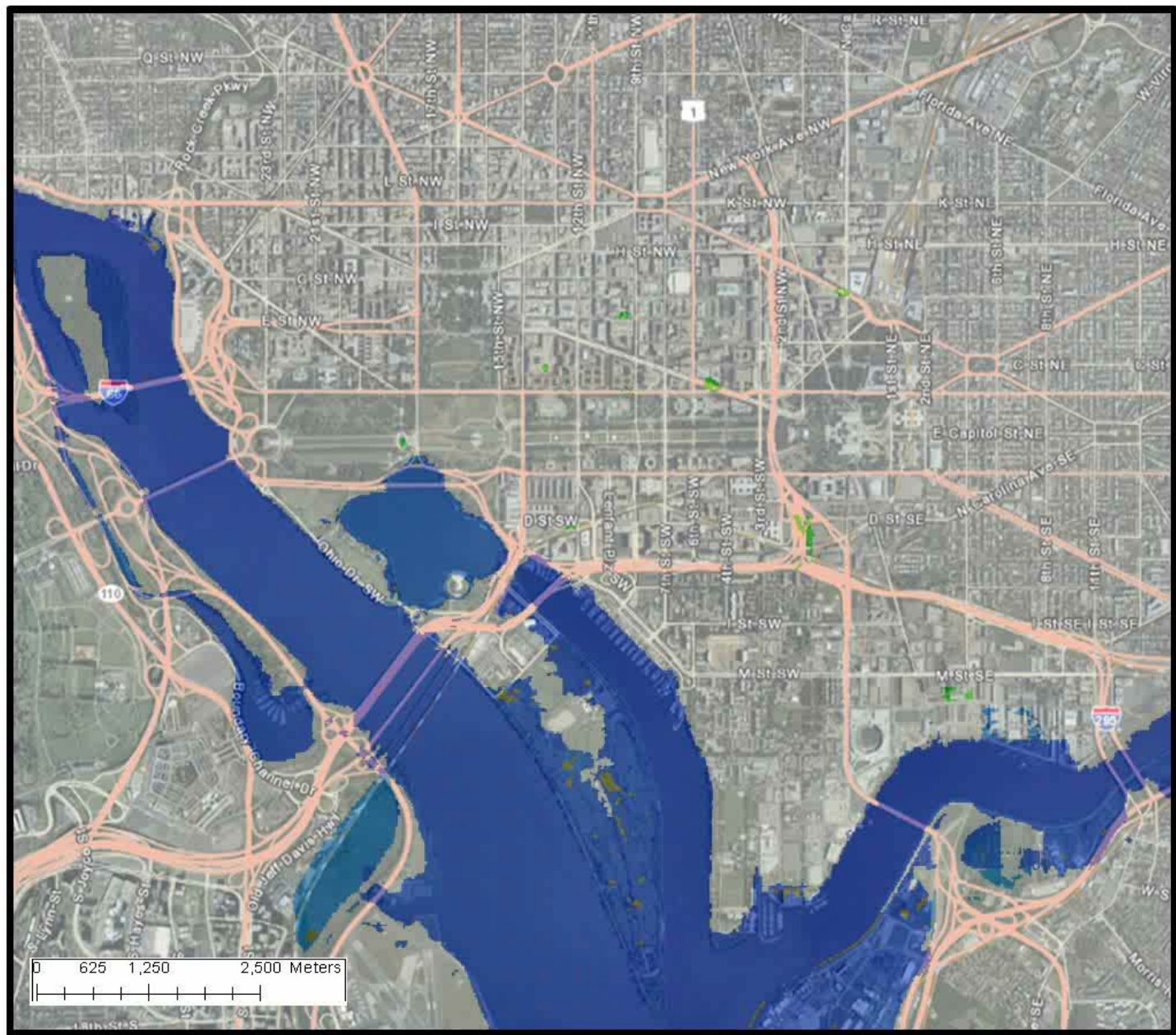
# Recreating 1936 Flood

Historical Observation



Model simulation results

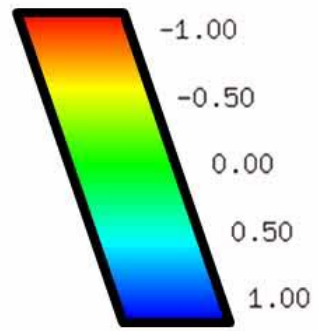


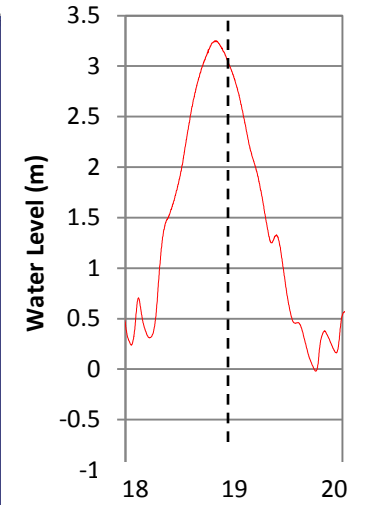
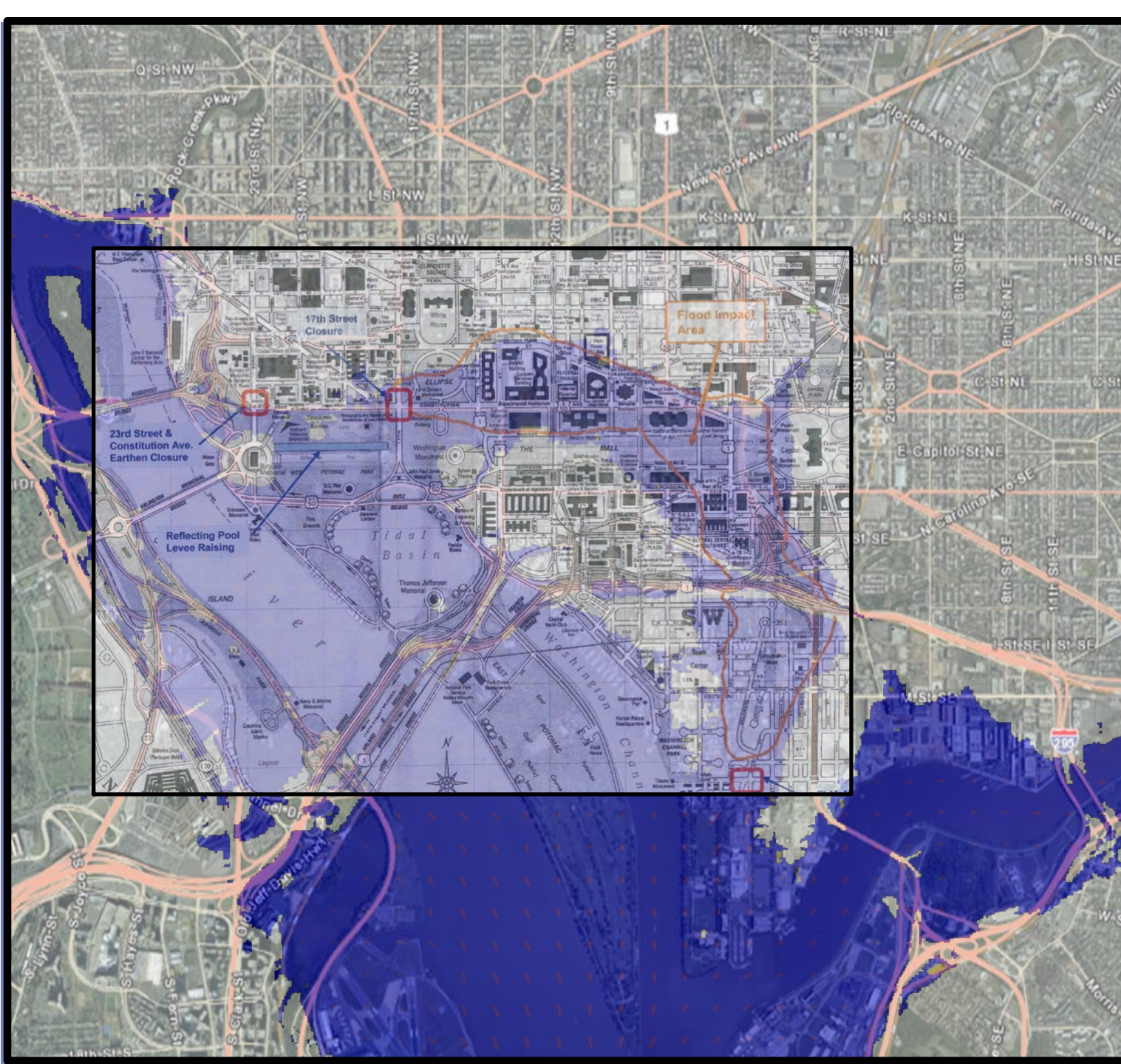


March 1936



Water Depth (m)

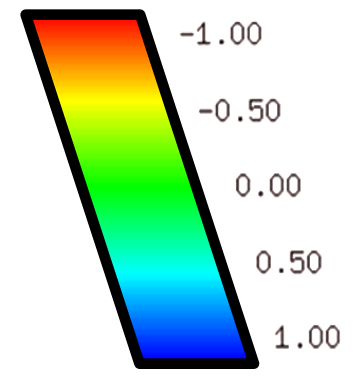


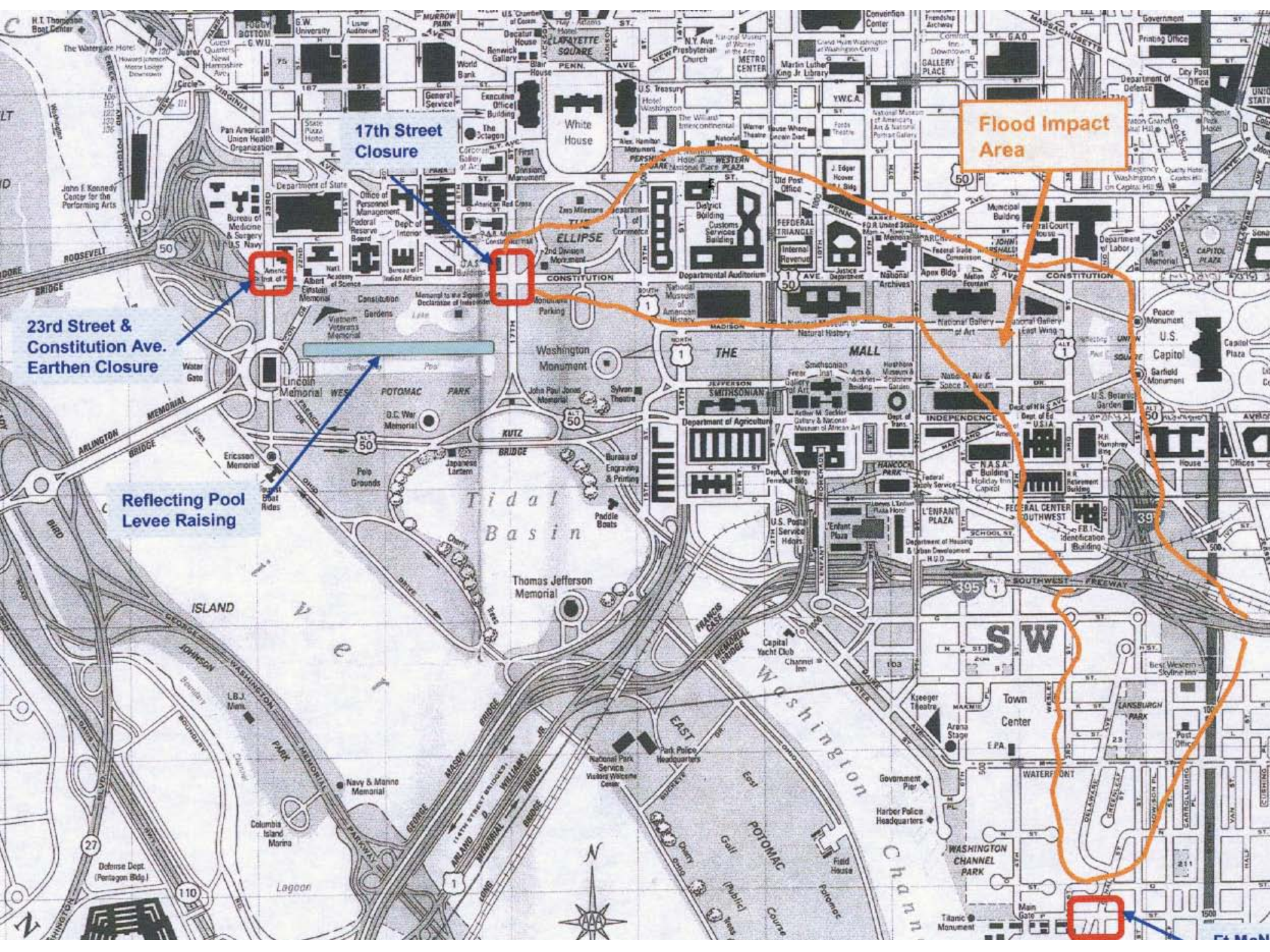


March 1936



Water Depth (m)





Flood Impact Area

17th Street Closure

23rd Street & Constitution Ave. Earthen Closure

Reflecting Pool Levee Raising

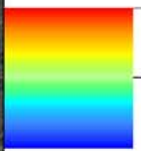
# Inundation in Old Town Alexandria, VA

Hurricane Isabel 2003 Sub-Grid Simulation



# Basin Bathymetry

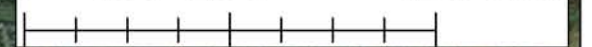
Value



High : 0

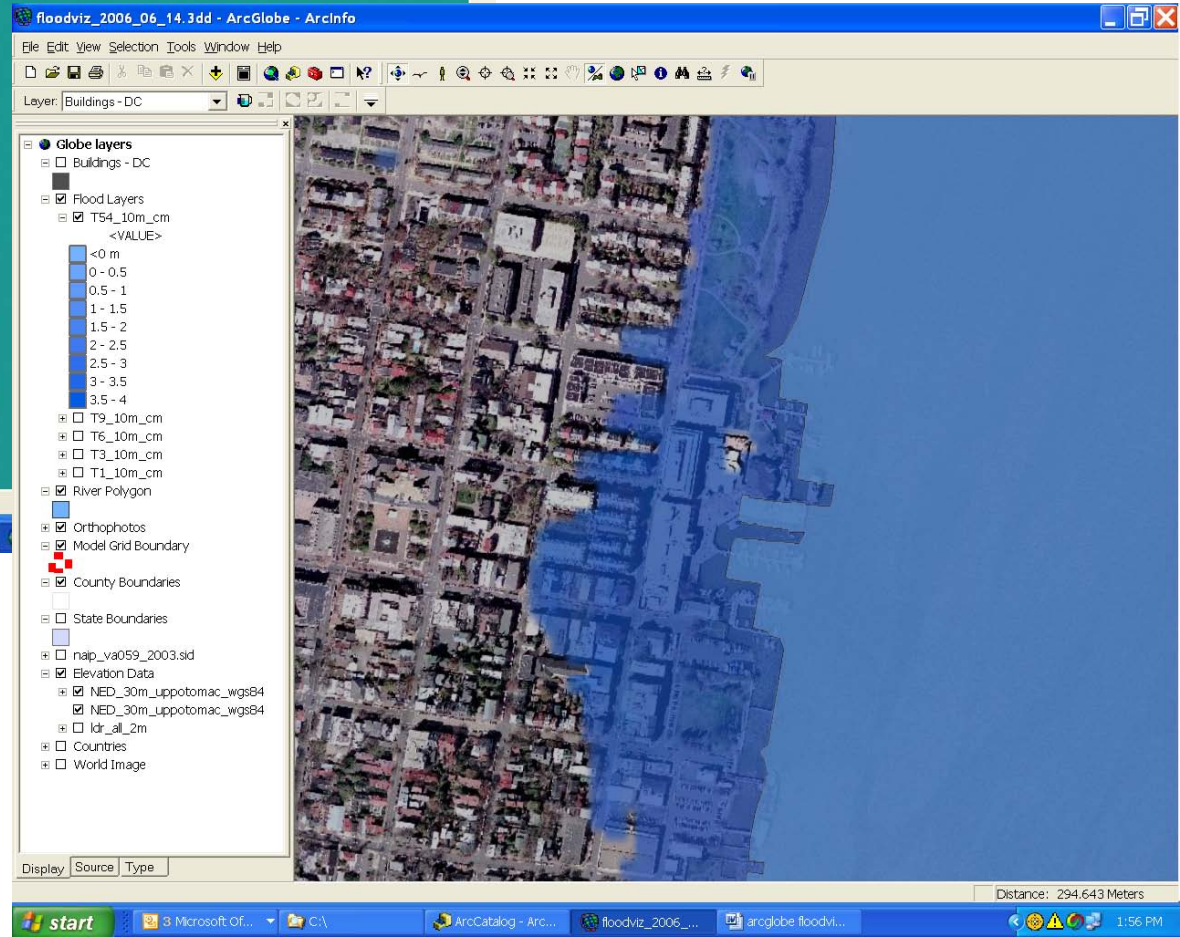
Low : -23.5886

0 1,000 2,000 4,000 Meters





← Sub-Grid Visualization  
 ← only 10% more  
 computation cost compared  
 with no sub-grid modeling



Fine Grid Visualization →  
 The computation cost is 5  
 times more using very fine  
 resolution -- 20 meter →



Figure 2: Alexandria Inundation-1936 Simulation. This figure demonstrates the inundation in the Alexandria area with 2005 topography responding to a 19 March 1936 scale flooding event. Within the Google Earth interface, one can zoom into street level or zoom out to watershed-scale overviews, or step through time during the evolution of the storm.

# Summary and future work

- Presently, the 10 m sub-grid can be run in a PC and finish 30 days simulation within a few hours
- If we use true fine grid, it will take 30 times more of the CPU time
- Further works:
  - (1) expand to 3D
  - (2) include salinity calculation

# What about sub-grid calculation for concentration in WQ model?

- So far what sub-grid modeling can achieve is to provide a set of rigorously derived high resolution velocities in the shallow water
- What do we go from here to the next step?
- My suggestion: Water quality and sediment dynamic modelers can consider taking the high resolution velocities as an input and work further for refining the sediment and SAV sub-model in the nearshore environment

